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Conference Abstract

Incorporating the Maximal Mean Power Profile in Time Trial Simulations for More Efficient Optimal Pacing Strategy Calculations

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Abstract: Mathematical modelling in cycling enables retrospective analysis and predictive simulations, crucial for optimizing performance. This study introduces a numerically efficient notation for incorporating a rider's maximal mean power profile, enhancing computational times for pacing strategy calculations while maintaining physiological relevance. Using exponentially weighted rolling averages (EWM) expedites MMP computation compared to classic averages. Applied to the 21st stage of the 2024 Tour de France, the methodology is adopted here in an optimal pacing strategy calculation with state-of-the-art complexity. The proposed solution offers a streamlined alternative to existing models, promising reduced computational costs and enhanced optimization algorithms, thus advancing cycling performance analysis.

Keywords: Dynamic Optimization, Mathematical Modelling, Exponentially Weighted Moving Averages.

1. Introduction

Mathematical modelling in cycling is widely used for retrospective analysis and/or predictive simulations of performance. Individual time trials are particularly suited for simulations, because drafting is not allowed, and riders are free to choose their own racing lines.

How the power output should be distributed along the course to minimize the final time, is the problem of the optimal pacing strategy calculation (Abbiss & Laursen, 2008). In mathematical terms, this translates into a dynamic optimization process. Nowadays, the most complex simulations involve 3D models of cycling dynamics and are solved by means of optimal control techniques (Zignoli & Biral, 2020). Pacing strategy calculations that can concomitantly consider environmental factors (such as wind and course geography) as well as rider's physiology are notoriously difficult to formulate and solve.

Here, a numerically efficient notation for the incorporation of a rider's maximal mean power profile into the simulation is introduced. It is believed that this notation can effectively improve computational times for pacing strategy calculations whilst preserving physiological meaning and wide applicability to real-world scenarios.

2. Materials and Methods

Computing the maximal mean power (MMP) profile is the practice of collecting the rolling averages of a cyclist's historical racing/training sessions and considering the maximum values for a range of window-lengths (Quod et al., 2010). Theoretically, there is no restriction to the kind of rolling average that can be used for the calculation of the MMP. Whilst the classic rolling average is commonly used, here the exponentially weighted (EWM) rolling averages are used. This choice can provide advantages: instead of requiring an entire window of power data to be computed, EWMs can be computed just by starting from the previous data point and the current power output point. Using the differential equation notation, a generic EWM of the power output can be computed during exercise as:

$$\tau \frac{dEWM_{\tau}}{dt} + EWM_{\tau} = P \qquad \text{Equation (1)}$$

Where τ represents the EWM time-characteristics, EWMτ represents the exponentially weighted average of the corresponding τ, and P is the mechanical power output. Because of the light formulation, calculating the MMP with a collection of EWMs is faster than using classic averages. In addition, classic averages require assumptions when data is not available for the entire window length. During a simulation, every EWMτ must be constrained below the maximum EWMτ available in the MMP (i.e. EWM_{MAX}).

Equation 1 is particularly best suited for optimal control problem formulations, as it basically just adds a state variable and a constraint to that variable (max(EWMτ)<$EWM\tau_{MAX}$ for each τ). The values of all EWMs are set to 0, i.e. meaning that the cyclist is starting the race in a fresh state.

The methodology of the pacing strategy calculation is applied here to the Individual Time Trial 21st Stage of the 2024 Tour de France (Monaco-Nice, 21st July 2024). The course was manually tracked using Google Earth, and then it was used in an optimal control framework as detailed in (Zignoli & Biral, 2020). In short, the algorithm was asked to find the solution that could lead to the minimum race time, while respecting the constraints dictated by the physics of the system (i.e. the equation of motion of the cyclist-bike system) and by the MMP profile. A set of equations resembling Equation 1, were introduce in the formulation for τ equal to 10'', 1', 3', 5', 6', 10', 20', and 30'. MMP was built using EWMs from a professional cyclist (CdA=0.19, CP=430) (Table 1).

Table 1. Values for the maximal exponentially weighted moving averages (EWM) adopted in the construction of the maximal mean power profile.

EWM	EWM_{MAX}
10 sec	1275 W
1 min	710 W
3 min	551 W
5 min	547 W
6 min	495 W
10 min	474 W
20 min	453 W
30 min	431 W

3. Results

The course is 35 km long. The optimal pacing solution was computed in 439 seconds on a Mac Book Pro (2015). The simulated power output for the optimal pacing strategy is reported in Figure 1, together with EWM_{10sec}, EWM_{10min}, and EWM_{10min}.

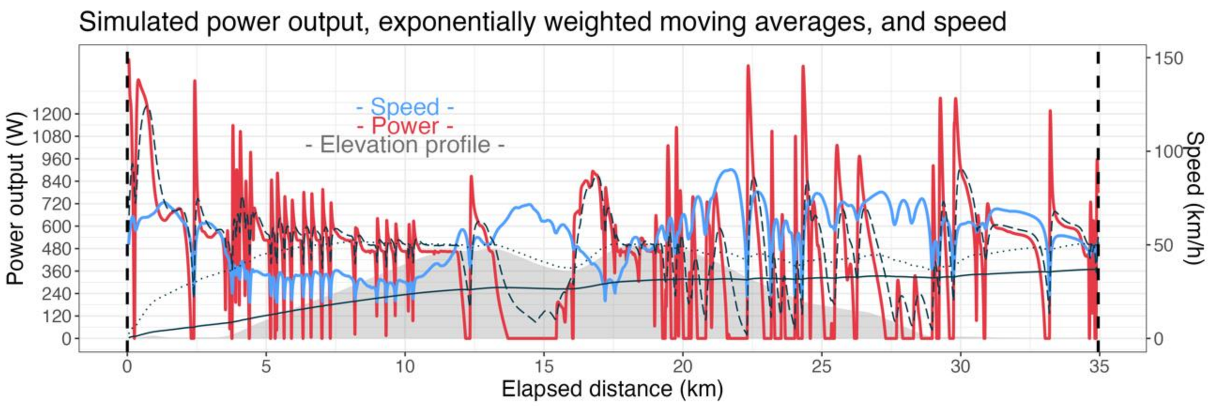

Figure 1. Predicted mechanical power output (red) for the optimal pacing strategy during the stage. Relevant exponentially weighted moving averages are reported also: EWM_{10sec} (dashed line), EWM_{10min} (dotted line), and EWM_{10min} (solid line).

The maximum values of the power output EWMs resulting from the simulation did not exceed the historical maximum EWMs reported in Table 1. The simulated maximum EWM values were: EWM_{10sec} = 1241 W, EWM_{1min} = 706 W, EWM_{3min} = 546, EWM_{5min} = 516 W, EWM_{6min} = 496 W, EWM_{10min} = 475 W, EWM_{20min} = 425 W, and EWM_{30min} = 369 W

4. Discussion

The solution proposed here can be used as a light alternative of the W'_{bal} model (broadly adopted in pacing strategy calculations), which is made complex by at least two factors (Skiba & Clarke, 2021): 1) there are ideally two different kinetics for W'_{bal} recovery and depletion, and 2) the variable W'_{bal} comes with an inferior constraint for W'_{bal} ($W'_{bal}>0$), but also for the power output ($P<CP$ when $W'_{bal}=0$), and a superior constraint for W'_{bal} ($W'_{bal}<W'$). Also, other models available of 'fatigue' and recovery require additional parameters to be calibrated (Fayazi et al., 2013), whilst the methodology introduced here solely relies on historical training and racing data to build the MMP. This, however, comes at the cost of including an arbitrary number of differential equations to the problem (i.e.: one for each $EWM\tau$ worth considering), and a constraint for every $EWM\tau$. The novelty of this methodology consists in the physiological constraints, that are directly included by means of the MMP profile points. The optimization algorithm is asked to solve the problem of the pacing strategy while being compliant with these constraints. The constraints are never violated during the simulation, so therefore the virtual cyclist is never pushing more than what is dictated by the MMP profile. The results are therefore deemed "physiologically possible" by definition, given that the EWMs of the power output are constrained by the EWM_{MAX} used to populate the MMP profile.

5. Practical Applications

While the complexity of the optimal pacing strategy formulation is recognized to be problem-specific, these initial findings are encouraging and suggest that the methodology presented here may effectively reduce computational costs and consequently enhance the optimization algorithm's convergence time to a solution. Furthermore, the only requirement is a reliable MMP, without the need to estimate fatigue-recovery kinetics as it is required with other models.

Supplementary Materials: The map of the cycling stage adopted for the simulations is available at this link.

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Conflicts of Interest: The methodology introduced here is adopted on the Athletica platform (athletica.ai). AZ holds stocks at Athletica.

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